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Turbulent buoyant convection from a source in a confined region

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An extension of the model first proposed by Baines & Turner (1969) is derived with careful attention to the conditions required for its application. The most important of these are that the Prandtl number ν/κ is of order unity or greater, the ratio of the length L to depth H of the region is greater than about 1·2 for the two-dimensional region considered and R is so large that $R \gtrsim L/H$ and $R \gg 1/\alpha$. The characteristic group

$$R \equiv \alpha^{\frac{2}{3}} F_0^{\frac{1}{3}} H/\nu.$$

R^2 is a Grashof number based on the source strength F_0 of the buoyant convection which is modelled using turbulent plume theory and the entrainment constant α, the ratio of inflow velocity across the edge of the plume to mean local plume velocity. The conditions on R ensure that the source of buoyant convection is the dominant transportive mechanism in the region and the restrictions on the aspect ratio ensure that there is clear separation between the passive motions in the bulk of the region and the intense highly confined buoyant convection.

The manner in which the convective fluid recirculates to become part of the passive interior is studied and shown to be controlled by the same dynamics as fluid intrusion into a stably stratified environment.

Several new solutions are obtained, including cases of steady conditions involving only one source.

1. Introduction

We here formulate a model system of convection at very high Rayleigh number driven by small steady buoyancy sources at the bounding horizontal surfaces. Attention will be focused on a two-dimensional symmetric region containing only one line-source of buoyancy, of strength $2F_0$, at say O on the lower boundary, where O is the origin of the x (horizontal) and z (vertically upwards) axes, with corresponding fluid velocities u and w. The three-dimensional axisymmetric problem is similar but will not be treated here. Only the part $x > 0$ will be considered and alternatively may be thought of as being bounded by fixed walls. As illustrated in figure 1 the source at O generates a plume of turbulent buoyant fluid which dilutes as it rises under gravity by entrainment of the interior fluid. Only the behaviour at large times is studied, that is at times much greater than the time scale for all the fluid in the confined region (hereafter referred to as the box) to have been entrained at least once into the plume, for then a quasi-steady state may be reached in which the velocities are everywhere steady but the buoyancy at every point increases with time.

Upon reaching the upper surface at $z = H$ the plume turns and spreads laterally in the outflow region. This is delineated from the interior where fluid flows towards the plume, by the surface $u = 0$. The turbulence in the region of outflow is quickly reduced towards the much lower level characteristic of the interior due to suppression by the stable stratification which is a feature of the interior region. The outflow fluid is merely interior fluid looked at at a slightly earlier time.

While all other boundaries of the box are assumed to be insulating, the horizontal surface $z = 0$ at the level of the buoyancy source is permitted to conduct buoyancy uniformly from the region at a rate $(1-\gamma_c) F_0$ ($|1-\gamma_c| < 1$), over the length L of the box. A diffusive boundary layer exists there and this will be shown to be thin.

Further, the fluid in the box is assumed to be gaining buoyancy at a uniform rate $\gamma_R F_0$. Negative γ_R would correspond to uniform loss of buoyancy due to, for example, long wave radiative divergence in the atmospheric boundary layer. Positive γ_R would correspond to a uniform buoyancy gain by, for example, internal heat generation by radioactive decay in the earth's mantle.

The most important simplification to be made in the model is the parameterization of the turbulent plume. The plume increases in volume flux and decreases in buoyancy with increasing distance by the turbulent entrainment of nearby fluid. Neither the actual mechanism of entrainment nor the rate at which it proceeds are fully understood (cf. Townsend 1970; Phillips 1972). It seems clear however that the rate of entrainment is, as Townsend states, 'set by the structure of the whole flow'. In situations where the plume motions satisfy similarity solutions the structure of the whole flow is directly related to the steady upward velocity characteristic of the plume at any level. For such cases Batchelor (1954) pointed out that the mean inflow velocity across the edge of the plume is proportional to the steady local upward velocity of the plume. This observation will be regarded as a basic assumption applicable to the present situation even though exact similarity forms are not obtainable. This is the 'entrainment assumption', and implies the same kind of turbulence structure and balance of forces at each level (Turner 1973, chap. 6; this reference contains further discussion of the assumption and its successful use in several situations).

The exact entrainment assumption to be used here is as follows: First the detailed ensemble-averaged structure of the plume is replaced by equivalent horizontal integral representations. Problems of definition of limits of integration inherent in this approach (see Kotsovinos & List 1977) are avoided by a novel step in the analysis (following Walin 1971) in which the plume is considered to overlie part of the interior. Then the horizontal inflow velocity from the interior to the plume at each level is assumed to be proportional to the steady upward velocity characteristic of the integral representation of the plume relative to vertical interior motions at the same level. The proportionality constant, α, is referred to as the entrainment parameter. Its value may be a weak function of the local Froude number of the plume as is suggested by a recent study of plane turbulent buoyant *jets* by Kotsovinos & List (1977). Without more information, however, there seems little point in complicating the analysis in view of the demonstrated success achieved in the present situation utilizing a constant α (Baines & Turner 1969; Baines 1975; Germeles 1975). A value close to 0·08 for α is appropriate here.

Now in a restricted formulation of no internal buoyancy generation, a non-diffusive interior and all surfaces non-conducting, Baines & Turner (1969, hereafter referred to

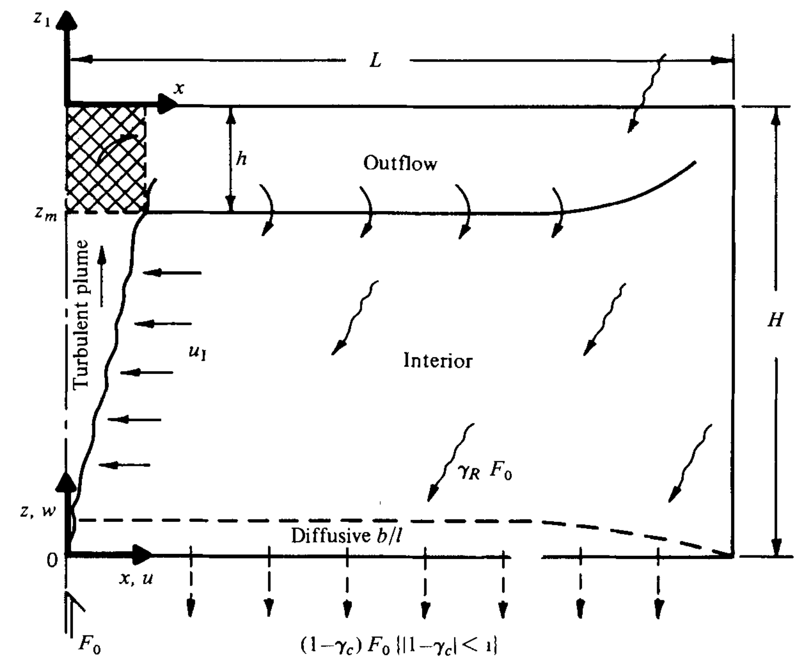

FIGURE 1. Definition sketch of the model; two-dimensional symmetry about $x = 0$ is implied.

as BT) first proposed the present problem in three-dimensional axisymmetric and two-dimensional symmetric geometries. Under assumptions which will be examined more carefully in the next section, they obtained large-time solutions for most properties of interest. Laboratory experiments confirmed predictions of the model and established the use of the entrainment assumption in this context. BT applied their results to explain in physical terms the once controversial 'counter-gradient' heat flux (Priestley 1959; Deardorff 1966) characteristic of clear-air convection in the lower atmosphere and also of the model problem of parallel plate convection at high Rayleigh numbers. Other applications have included the cooling of a room by a strip cooler, the heating of the atmosphere below the cloud-base, and the cooling of the upper mixed-layer in the ocean.

In extending the BT model Baines (1975) has relaxed the condition of an insulating upper surface to the box to permit entrainment of overlying light fluid by the turning plume (the hatched region in figure 1), thus allowing more realistic modelling of geophysical problems. For convenience of presentation the present model does not permit this extension although it is straightforward. An extra source of buoyancy flux to the interior must be included and conditions must be imposed on the strength of the density jump at the upper surface to ensure that the plume remains within the box.

Germeles (1975) has also extended the work of BT to include forced plumes from nozzles which may be off-centred and inclined and has applied the work to the filling of LNG storage tanks. The criticisms of Kotsovinos & List (1977) are germane to Germeles' approach and should be considered in similar extensions.

The generality of the present model allows its application to new problems as well as to the above with weaker restrictions applied. Thus for example the inclusion of radiation of buoyancy permits study of simple models of convection in the earth's mantle and of the circulation in enclosures with intense localized heating from below. With a diffusive interior and conducting lower boundary a case of a true steady state with only one buoyancy source is possible. This case has been applied in an inverted realization to the deep circulation of the Red Sea (Manins 1973*a*). An extension of the model to include a non-uniform buoyancy distribution on the lower boundary to supply the turbulent plume in the confined region has also been proposed (Killworth & Manins 1979).

Finally, consider the parameter range for validity of the model. It will be shown to be valid when the aspect ratio $A \equiv L/H$ and Prandtl number $\sigma \equiv \nu/\kappa$ are fixed and greater than unity (possibly large) and the Grashof number (and hence the Rayleigh number) based on the depth and plume source strength is large, tending to infinity. These are the conditions for a passive interior and dominant boundary layer (the plume) and are the same as required for similar behaviour in convection between differentially heated parallel plates, no matter whether these are horizontal (e.g. Moore & Weiss 1973; Wesseling 1969) or vertical (Gill 1966; Cormack, Leal & Imberger 1974; Imberger 1974). The entrainment parameter α is the natural ordering parameter here since it indicates the relative strengths of interior and plume motions (as well as the ratio of the scale thickness of plume to the height of the box) and this ratio must be small for the desired circulation to exist.

2. The equations of motion for the confined plume model

If at an arbitrarily chosen time a reference density of the fluid in the box is ρ_1 and the buoyancy at a point where the density is ρ is defined by

$$\Delta = -g(\rho-\rho_1)/\rho_1, \tag{2.1}$$

then to the Boussinesq approximation (Gray & Griorgini 1976), the governing differential equations for the two-dimensional flow perturbation from the hydrostatic field characterized by ρ_1 are

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z} = -\frac{1}{\rho_1}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right),\\
&\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z} = -\frac{1}{\rho_1}\frac{\partial p}{\partial z}+\Delta+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right),\\
&\frac{\partial \Delta}{\partial t}+u\frac{\partial \Delta}{\partial x}+w\frac{\partial \Delta}{\partial z} = \kappa\left(\frac{\partial^2 \Delta}{\partial x^2}+\frac{\partial^2 \Delta}{\partial z^2}\right)+\gamma_R\frac{F_0}{LH},\\
&\partial u/\partial x+\partial w/\partial z = 0.
\end{aligned}\right\} \tag{2.2}$$

Here p is the perturbation pressure and ν and κ are diffusion coefficients of momentum and buoyancy. The boundary conditions on the surfaces of the box containing the confined turbulent plume (figure 1) are

$$\left.\begin{aligned}
&w = 0, \quad \partial\Delta/\partial z = 0 \quad \text{on} \quad z = H,\\
&u = 0, \quad \partial\Delta/\partial x = 0 \quad \text{on} \quad x = L,\\
&u = w = 0, \quad \int_0^L \kappa\frac{\partial\Delta}{\partial z}\,dx = (1-\gamma_c)F_0, \quad \int_0^L \delta(x)\,\Delta w\,dx = F_0 \quad \text{on} \quad z = 0,
\end{aligned}\right\} \tag{2.3}$$

$x = 0$ is a line of symmetry.

Here $\delta(x)$ is the delta function and the conditions on $z = 0$ specify the rate of buoyancy diffusion through the boundary and the strength of the plume.

Since by assertion the turbulent plume is the prime mover of the system it is clear that the greatest velocity and buoyancy scales in the box are those characterizing the behaviour of the plume. Subscript I denotes a variable representing the properties of the interior, subscript P a variable of the plume region, and superscript P a variable representing the properties of the plume region measured relative to the interior. Then the solution for the plume region is partitioned as follows:

$$\phi_P = \phi_I + \phi^P, \tag{2.4}$$

where ϕ is any one of the dependent variables and ϕ^P is assumed negligibly small everywhere except in the thin turbulent shear layer characterizing the plume. While varying with the interior scale H along the plume, ϕ^P is thus assumed to vary much faster perpendicular to the plume. This variation is described by a characteristic plume half-width b, $\ll H$, so that $\phi^P = 0$ as $x/b \to \infty$.

The entrainment assumption (§1) may be written to a first approximation [see (2.23) below for an exact statement] as

$$-u_I = \alpha w^P \quad \text{as} \quad x \to 0. \tag{2.5}$$

Then analysis (Manins 1973*b*) suggests the following non-dimensionalization results in an adequate normalization of the variables:

$$\left.\begin{aligned} &(x_I, x_p, z) = H(A\xi^*, \alpha\chi^*, \zeta^*); \quad t = \alpha^{-\frac{2}{3}} F_0^{-\frac{1}{3}} L\tau^*; \quad p/\rho_1 = \alpha^{-\frac{2}{3}} F_0^{\frac{2}{3}} p^*; \\ &\Delta = \alpha^{-\frac{2}{3}} F_0^{\frac{2}{3}} H^{-1}\Delta^*; \quad (u, w_I, w_p) = \alpha^{\frac{2}{3}} F_0^{\frac{1}{3}}(u^*, w_I^*/A, w_p^*/\alpha); \end{aligned}\right\} \tag{2.6}$$

where $A \equiv L/H$ is the aspect ratio of the box, and * denotes a non-dimensionalized variable. The normalization for the outflow region is discussed in §2.3.

The separate regions of interior, plume and outflow are now considered in turn.

2.1. *The interior*

The equations of motion (2.2) are written in terms of dimensionless interior variables. Then, dropping the asterisks, there results:

$$\begin{gathered} \alpha^2 \left\{\frac{\partial u_I}{\partial \tau} + u_I \frac{\partial u_I}{\partial \xi} + w_I \frac{\partial u_I}{\partial \zeta}\right\} = -\frac{\partial p_I}{\partial \xi} + \frac{\alpha^2 A}{R}\left\{\frac{\partial^2 u_I}{\partial \xi^2}\Big/ A^2 + \frac{\partial^2 u_I}{\partial \zeta^2}\right\}; \\ \frac{\alpha^2}{A^2}\left\{\frac{\partial w_I}{\partial \tau} + u_I \frac{\partial w_I}{\partial \xi} + w_I \frac{\partial w_I}{\partial \zeta}\right\} = -\frac{\partial p_I}{\partial \zeta} + \Delta_I + \frac{\alpha^2}{AR}\left\{\frac{\partial^2 w_I}{\partial \xi^2}\Big/ A^2 + \frac{\partial^2 w_I}{\partial \zeta^2}\right\}; \\ \frac{\partial \Delta_I}{\partial \tau} + u_I \frac{\partial \Delta_I}{\partial \xi} + w_I \frac{\partial \Delta_I}{\partial \zeta} = \frac{A}{\sigma R}\left\{\frac{\partial^2 \Delta_I}{\partial \xi^2}\Big/ A^2 + \frac{\partial^2 \Delta_I}{\partial \zeta^2}\right\} + \gamma_R; \\ \frac{\partial u_I}{\partial \xi} + \frac{\partial w_I}{\partial \zeta} = 0; \end{gathered} \tag{2.7}$$

where $\sigma \equiv \nu/\kappa$ is the Prandtl number, and $R \equiv \alpha^{\frac{2}{3}} F_0^{\frac{1}{3}} H/\nu$ is a characteristic Reynolds number (or square root of a Grashof number) of the interior (and of the plume). Then subject to the conditions

$$R/A \gtrsim 1 \gg \alpha^2, \quad A \gtrsim 1, \quad \sigma \gtrsim 1 \tag{2.8}$$

the equations may be expanded in terms of the small parameter α^2 as follows. Setting

$$\phi_I = \phi_0 + \alpha^2\phi_1 + \alpha^4\phi_2 + \ldots;$$

then the zeroth-order equations obtained from (2.7) by substitution of this expansion and collecting terms in powers of α^2 are

$$0 = -\partial p_0/\partial \xi, \quad 0 = -\partial p_0/\partial \xi + \Delta_0, \tag{2.9a, b}$$

which imply $\Delta_0 \equiv \Delta_0(\tau, \zeta)$ only, so

$$\frac{\partial \Delta_0}{\partial \tau} + w_0 \frac{\partial \Delta_0}{\partial \zeta} = \frac{1}{J}\frac{\partial^2 \Delta_0}{\partial \zeta^2} + \gamma_R \tag{2.9c}$$

which implies $w_0 \equiv w_0(\tau, \zeta)$, and

$$\frac{\partial u_0}{\partial \xi} + \frac{\partial w_0}{\partial \zeta} = 0. \tag{2.9d}$$

Here $J \equiv \alpha^{\frac{4}{3}} F_0^{\frac{1}{3}} H^2/\kappa L = R\sigma/A$ expresses the relative importance of advection processes compared to diffusion processes in the box. J will be greater than unity in practice as free convection situations are primarily advective (Stommel 1962). Further, equations (2.9) show that to lowest order the interior isopycnals are horizontal and vertical motion is independent of position along any isopycnal.

Now the entrainment assumption (2.5) is in dimensionless terms

$$-u_0 = w^P \quad \text{as} \quad \xi \to 0, \tag{2.10}$$

so from (2.9) it follows that

$$u_0 = u_0|_{\xi=0}(1-\xi) = -w^P(1-\xi) \tag{2.11}$$

and

$$w_0 = \int^{\zeta} u_0|_{\xi=0}\, d\zeta' = -\int^{\zeta} w^P\, d\zeta', \tag{2.12}$$

where $u_0|_{\xi=0}$ is u_0 evaluated at $\xi = 0$.

It can be seen from the solution (2.11) that to lowest order the horizontal velocity in the interior increases linearly from zero at the far wall of the box to a maximum at the plume edge where the interior fluid is incorporated into the plume by entrainment. The interior is also subject to a sinking motion (2.12) just sufficient to make up the loss by entrainment at any level. It will be found (§ 3) that in all cases w^P is only a weakly decreasing function of increasing ζ so to a first approximation the interior stream-function defined such that

$$u_0 = \frac{\partial \psi_I}{\partial \zeta} \quad \text{and} \quad w_0 = -\frac{\partial \psi_I}{\partial \xi}$$

may be written as

$$\psi_I = \text{const} \times \zeta(1-\xi). \tag{2.13}$$

Thus the interior flow to lowest order is an elementary stagnation flow with stagnation point at the lower right-hand corner of the box in figure 1.

The first-order interior equations obtained from (2.7) subject to (2.8) are

$$\left.\begin{aligned}
&\frac{\partial u_0}{\partial \tau}+u_0\frac{\partial u_0}{\partial \xi}+w_0\frac{\partial u_0}{\partial \zeta} = -\frac{\partial p_1}{\partial \xi}+O(A/R),\\
&\frac{1}{A^2}\left\{\frac{\partial w_0}{\partial \tau}+w_0\frac{\partial w_0}{\partial \zeta}\right\} = -\frac{\partial p_1}{\partial \zeta}+\Delta_1+O(1/AR),\\
&\frac{\partial \Delta_1}{\partial \tau}+u_0\frac{\partial \Delta_1}{\partial \xi}+w_0\frac{\partial \Delta_1}{\partial \zeta}+w_1\frac{\partial \Delta_0}{\partial \zeta} = \frac{1}{J}\left[\frac{\partial^2 \Delta_1}{\partial \xi^2}\Big/A^2+\frac{\partial^2\Delta_1}{\partial \zeta^2}\right]
\end{aligned}\right\} \qquad (2.14)$$

and

$$\frac{\partial w_1}{\partial \xi}+\frac{\partial w_1}{\partial \zeta} = 0.$$

If $A^2 \approx 1$ then from (2.14*a, b*) the interior region would be characterized by vertical accelerations comparable to the buoyancy and horizontal accelerations. Further, from (2.6) the vertical and horizontal velocities would be comparable. The result for the interior would be an 'overturning' type of behaviour as observed by *BT* when they varied the aspect ratio of their box to near unity. This is not the behaviour sought here so the restriction on A must be strengthened to $A^2 \gg 1$. Laboratory experiments (Manins 1973*b*) show that $A^2 > 1{\cdot}5$ is sufficient in practice.

Just as for the zeroth-order equations (2.9), the diffusion of buoyancy becomes important in equation (2.14*c*) on a depth scale $\zeta \lesssim 1/J^{\frac{1}{2}}$. A boundary layer of thickness $\delta = H/J^{\frac{1}{2}} = (\kappa\alpha^{-\frac{2}{3}}F_0^{-\frac{1}{3}}L)^{\frac{1}{2}}$, independent of the depth of the box, exists adjacent to the surface from which the turbulent plume rises. A thin buoyancy boundary layer, analogous to an Ekman layer in rotating flow problems, is also present on the wall $\xi = 1$ but will not be considered here (see Walin 1971 for further discussion).

Although the solutions (2.11) and (2.12) to the zeroth-order equations will be assumed to describe the interior to sufficient accuracy, it may be further noted that from (2.14) the first-order horizontal pressure gradient for steady flow may be written as

$$\frac{\partial p_1}{\partial \xi} = \left[(w^P)^2-\int^{\zeta} w^P d\zeta'\,\frac{\partial w^P}{\partial \zeta}\right](1-\xi)+O(A/R).$$

Now $\partial w^P/\partial\zeta < 0$ for all ζ in the region of the interior so $\partial p_1/\partial\xi > 0$ in the same range. Thus the maximum horizontal pressure gradient in the interior is in the vicinity of the plume and is consistent with the plume being confined to as narrow a neighbourhood of $\xi = 0$ as possible.

2.2. *The turbulent plume*

Now the equations of motion (2.2) for the plume may be written symbolically as (Walin 1971) $\mathscr{P}(\phi_P) = 0$. By definition

$$\mathscr{P}(\phi_I) = \mathscr{P}(\phi_I+\phi^P) = 0$$

so

$$\mathscr{P}(\phi_I+\phi^P)-\mathscr{P}(\phi_I) = 0 \qquad (2.15)$$

represents the equations of motion of the plume *relative* to interior motions.

Relative plume variables (ϕ^P) are represented as the sum of their ensemble mean (ϕ^P) and turbulent fluctuating parts (ϕ')

$$\phi^P = \widetilde{\phi^P}+\phi'.$$

Next, ensemble averages of equations (2.15) are taken. The plume is driven by the buoyancy Δ^P to a mean velocity $\widetilde{w^P}$ so the Reynolds stress terms which result from the averaging process must always influence but never dominate the plume motion. This implies $\widetilde{u'w'} \lesssim \alpha(\widetilde{w^P})^2$, $\widetilde{u'\Delta'} \lesssim \alpha\widetilde{w^P}\widetilde{\Delta^P}$ and $\widetilde{w'\Delta'} \lesssim \widetilde{w^P}\widetilde{\Delta^P}$ in the plume. Then subject to further requirements which ensure diffusion is small, namely

$$1 \gg \alpha, \quad R \gg 1/\alpha, \quad \sigma \gtrsim 1, \tag{2.16}$$

the plume equations [and hence the interior equations (2.9a, b)] are steady to $O(\alpha/A)$ and, omitting the horizontal momentum equations, become

$$\left.\begin{aligned} &\widetilde{u^P}\frac{\partial\widetilde{w^P}}{\partial\xi} + \widetilde{w^P}\frac{\partial\widetilde{w^P}}{\partial\zeta} + u_I\frac{\partial\widetilde{w^P}}{\partial\xi} + \frac{\partial}{\partial\xi}\widetilde{u'w'} = \widetilde{\Delta^P}, \\ &\widetilde{u^P}\frac{\partial\widetilde{\Delta^P}}{\partial\xi} + \widetilde{w^P}\frac{\partial\widetilde{\Delta^P}}{\partial\zeta} + u_I\frac{\partial\widetilde{\Delta^P}}{\partial\xi} + \widetilde{w^P}\frac{\partial\Delta_I}{\partial\zeta} + \frac{\partial}{\partial\xi}\widetilde{u'\Delta'} + \frac{\partial}{\partial\zeta}\widetilde{w'\Delta'} = 0 \\ \text{and} \quad &\frac{\partial\widetilde{u^P}}{\partial\xi} + \frac{\partial\widetilde{w^P}}{\partial\zeta} + 0. \end{aligned}\right\} \tag{2.17}$$

The buoyancy source condition from (2.3) on $\zeta = 0$ becomes to lowest order

$$\int_0^{\chi\to\infty} \delta(\xi)\,[\widetilde{w^P}\widetilde{\Delta^P} + \widetilde{w'\Delta'}]\,d\xi = 1 \quad \text{at} \quad \zeta = 0 \tag{2.18}$$

and the definition of ϕ^P gives

$$\lim_{\chi\to\pm\infty} \{\widetilde{u^P}, \widetilde{w^P}, \widetilde{\Delta^P}, \widetilde{u'w'}, \widetilde{u'\Delta'}, \widetilde{w'\Delta'}\} = 0, \tag{2.19}$$

where, from (2.6), $\chi = A\xi/\alpha$.

Now the results of Kotsovinos & List (1977) show that as much as 40% of the buoyancy flux $w^P\Delta^P$ is by the turbulent flux $\widetilde{w'\Delta'}$ in a free plume. A similarity state is implied in which $\widetilde{w'\Delta'}/\widetilde{w^P}\widetilde{\Delta^P}$ is a constant. For the present problem of a plume in a confined region a state of similarity for the buoyancy flux is assumed. The equations (2.17) are integrated across the width of the plume and (2.19) imposed. 'Top hat' values $\overline{w}^P$ and $\overline{\Delta}^P$ and dimensionless plume half-width b (of dimensional magnitude αH) are defined by

$$\left.\begin{aligned} &\int_0^{\chi\to\infty} w^P d\xi = \overline{w}^P b, \quad \int_0^{\chi\to\infty} \widetilde{(w^P)^2}\,d\xi = (\overline{w}^P)^2 b, \\ &\int^{\chi\to\infty} \widetilde{\Delta^P}\,d\xi = \overline{\Delta}^P b, \quad \int_0^{\chi\to 0} (\widetilde{w^P}\widetilde{\Delta^P} + \widetilde{w'\Delta'})\,d\xi = \lambda\overline{w}^P\overline{\Delta}^P b. \end{aligned}\right\} \tag{2.20}$$

The parameter λ defined by (2.20d) expresses the above similarity assumption, as well as allowing for the observed slight difference in horizontal extent of the buoyancy and velocity fields (e.g. Rouse, Yih & Humphreys 1952). For a 60% contribution to the buoyancy flux by mean transport, $\lambda \approx 1{\cdot}7$.

Henceforth the overbar will be omitted from the scale values defined by (2.20), and upon integrating (2.17) the mean vertical momentum equation for the plume from (2.17 *a*) becomes

$$\frac{d}{d\zeta}[b(w^P)^2] = b\Delta^P. \tag{2.21}$$

The buoyancy equation (2.17 *b*) becomes

$$\frac{d}{d\zeta}(\lambda b w^P \Delta^P) = -b w^P \frac{\partial \Delta_I}{\partial \zeta} \tag{2.22}$$

and continuity (2.17 *c*) using (2.10) is now

$$\frac{d}{d\zeta}(bw^P) = -u_I\Big|_{\substack{\xi\to 0\\ \chi\to\infty}} = w^P. \tag{2.23}$$

The latter part of (2.23) is the mathematical statement of the entrainment assumption described in § 1.

The boundary condition (2.18) for the plume becomes

$$bw^P = b(w^P)^2 = 0, \quad \lambda b w^P \Delta^P = 1 \quad \text{at} \quad \zeta = 0. \tag{2.24}$$

The equations (2.21)–(2.23) with (2.24) are similar to the plume equations derived by Morton, Taylor & Turner (1956) and used by BT. The major difference is that here the dependent variables are all relative to the corresponding variables in the interior, thus permitting a variety of interior motions to be incorporated.

2.3. *The region of outflow*

The details of the outflow are of secondary importance so long as the region occupies only a small portion of the box and the time scale for the outflow to distribute fluid to correctly supply the interior is a small fraction of the time scale of the interior. The main purpose here is to show that these conditions are met albeit not strongly.

Well-mixed turbulent fluid spreads laterally from the turning plume (hatched region in figure 1) driven by the pressure gradient due in part to the small buoyancy excess of this fluid relative to that near the far end and interior. Interfacial shear stresses between the counter-flowing outflow and interior bring the fluid there to rest in the horizontal direction. At the same time the isopycnals in the outflow are displaced laterally and downwards by subsequent outflow fluid of greater buoyancy and a small discontinuity in the buoyancy occurs at the interface (the surface $u = 0$) close to the plume only. Since the time scale for lateral outflow will be shown to be significantly shorter than the interior timescale the isopycnals are caused to become increasingly tilted towards horizontal, further aided by the fact that fluid displaced into the interior encounters flow back towards the plume. In this way the outflow merges with the interior fluid to satisfy the solutions (2.11), (2.12).

For convenience a new co-ordinate system (x, z_1) is defined with the same orientation but centred at the beginning of the outflow region, as shown in figure 1. The motions in this region must be steady since both the plume and interior regions are steady.

Then denoting by subscript T a variable in the thin outflow of thickness h, the boundary layer equations downstream of the turning region are

$$\left.\begin{aligned}\frac{\partial}{\partial x}u_T^2+\frac{\partial}{\partial z_1}w_T u_T &= -\frac{\partial p_T}{\partial x}+\frac{\partial}{\partial z_1}\left(K\frac{\partial u_T}{\partial z_1}\right)\\ 0 &= -\partial p_T/\partial z_1+\Delta_T.\end{aligned}\right\} \qquad (2.25)$$

The turbulent shear stress in (2.25a) has been parameterized in terms of an eddy-diffusion coefficient K. Integrating equations (2.25) with respect to z_1 and utilizing (2.9a, b) gives for the horizontal momentum equation

$$\frac{\partial}{\partial x}\left[\int_{-h}^{0}u_T^2\,dz_1-\int_{-h}^{0}z_1(\Delta_T-\Delta_{Im})\,dz_1\right]=\int_{-h}^{0}\frac{\partial}{\partial z_1}\left(K\frac{\partial u_T}{\partial z_1}\right)dz_1, \qquad (2.26)$$

where subscript m indicates evaluation at $z_1 = -h$, that is $z = z_m$ (figure 1).

Outflow scales U, $\bar{\Delta}$ are defined by

$$U^2h=\int_{-h}^{0}u_T^2\,dz_1,\quad \bar{\Delta}h^2/2=\int_{-h}^{0}z_1(\Delta_T-\Delta_{Im})\,dz_1, \qquad (2.27)$$

and it is noted that Δ_T near the plume equals $\Delta_{Pm} = \Delta_m^P+\Delta_{Im}$ since the fluid in the turning region is well mixed. Thus $\bar{\Delta}$ is the buoyancy difference near the plume between the outflow and interior. Then up to distances downstream in the outflow where the right-hand side of (2.26) becomes significant, (2.26) may be written as

$$Fr^2 \equiv U^2/(\bar{\Delta}h) \approx \tfrac{1}{2}. \qquad (2.28)$$

That is, the internal Froude number of the outflow is near unity.

A close analogy may be drawn between the behaviour of the outflow in this problem and that of the intrusion of homogeneous fluid at constant rate and high R into a stably stratified environment (Manins 1976; Imberger, Thompson & Fandry 1976). In that problem the speed and thickness of the intrusion is set predominantly by the local buoyancy–inertia dynamics and is not simply related to dimensions of the source. Here too it may be expected that the outflow thickness is locally determined and will be constant for some significant distance downstream. Laboratory experiments have confirmed this is so (Manins 1973b): departure from constant thickness occurred only over the latter 40 % of the range of x.

This description of the outflow in terms of a unique internal Froude number is, as was found for the case of intrusions at high R (Manins 1976), an oversimplification of a closely coupled system. However, for the purposes of deriving a time scale, T, and thickness, h, it is assumed that (2.28) is adequate and for ease of presentation results for the case of a practically non-diffusive fluid are anticipated (§ 3.1). Since then (3.5) $bw^P\Delta^P = (1-\zeta)F_0/\lambda$, at $z_1 = -h$

$$q^P\Delta_m^P \equiv bw^P.\Delta_m^P=\frac{F_0}{\lambda}\frac{h}{H}. \qquad (2.29)$$

It is noted that near the plume $Uh \approx bw^P|_{-h}$ and it follows from (2.6), (2.28) and (2.29) that

$$h/H = \lambda^{\frac{1}{3}}(\alpha/Fr)^{\frac{1}{2}}\,(q_m^{P*})^{\frac{2}{3}} \qquad (2.30)$$

and

$$T_T/T_I = u_I/U = \lambda^{\frac{1}{3}}(\alpha/Fr)^{\frac{1}{2}}\,(q_m^{P*})^{\frac{2}{3}}/b_m^*. \qquad (2.31)$$

With $\lambda = 1, \alpha = 0{\cdot}08$ and $Fr = 2^{-\frac{1}{2}}$ it follows for this case $h/H \approx 0{\cdot}25$, $T_T/T_I \approx 0{\cdot}26$. Thus the outflow occupies about 25 % of the box and the time scale for the residence of plume fluid in the interior is some four times larger than that of the outflow. Laboratory experiments (Manins 1973*b*) support these findings.

If it were merely the case that mean kinetic energy was conserved in the turning region, the plume would turn in its own width so near the plume $h \approx b_m$ and $U \approx w_m^P$. Since $b_m \approx \alpha z_m$, $h/H \approx \alpha/(1+\alpha)$ which is a much smaller ratio than given by (2.30). But the internal Froude number in this case is

$$Fr^2 = \frac{U^2}{\Delta h} \approx \frac{(w_m^P)^2}{\Delta_m^P b_m} \approx \frac{\lambda}{F_0}\frac{H}{h}(w_m^P)^3$$

$$\approx \lambda \frac{1+\alpha}{\alpha^2}(w_m^{P*})^3 \gg 1.$$

Thus the outflow entrains fluid from the interior until the local internal Froude number (a function of position downstream) drops to a value determined by a downstream 'control' (Wilkinson & Wood 1971). Now the 'density jump' which must form immediately downstream of the turning plume is unstable because the downstream control is a wall ($x = L$). The jump must 'flood', the layer thickens and entrainment ceases. The flow is then described by the previous analysis so the outflow scales are (2.29) and (2.30).

3. Solutions for the asymptotic state of large time

The buoyancy flux into the box is steady and

$$\iint_{\text{box}} \frac{\partial \Delta}{\partial t}\,dx\,dz = F_0 - (1-\gamma_c)\,F_0 + \gamma_R F_0, \tag{3.1}$$

as is clear from figure 1. In the asymptotic state of large time the buoyancy at every point increases linearly with time. Consider the class of solutions to (3.1) in which $\partial\Delta/\partial t$ is independent of position so that buoyancy at every point increases linearly with time *at the same rate* and the velocities are steady.

Then in particular, in dimensionless variables again,

$$\partial\Delta_I/\partial\tau = \gamma_c + \gamma_R. \tag{3.2}$$

Equation (3.2) shows that there are three different steady solutions to the problem of turbulent buoyant convection from a single source in a confined region:

(*a*) $\gamma_c = \gamma_R = 0$, a diffusive, steady system;

(*b*) $\gamma_c = -\gamma_R \lessgtr 1$, a diffusive, radiating steady system;

(*c*) $\gamma_c = -\gamma_R = 1$, a non-diffusive, radiating steady system.

Other solutions involve non-zero $\partial\Delta_I/\partial\tau$ so then the interior buoyancy varies linearly with time. If equal and opposite sources may exist in the box then a further set of steady solutions is obtainable. One such case was considered in *BT* as a model of parallel plate convection at high Rayleigh number.

Now in view of (2.4), continuity of volume flux across any horizontal surface below the outflow gives

$$q^P \equiv bw^P = -w_I, \tag{3.3}$$

and using (3.2) and (3.3) the buoyancy equation for the interior (2.9*c*) becomes

$$-bw^P \frac{\partial \Delta_I}{\partial \zeta} = \frac{1}{J}\frac{\partial^2 \Delta_I}{\partial \zeta^2} - \gamma_c. \tag{3.4}$$

If now the buoyancy flux equation (2.22) for the plume is substituted into (3.4) and the result integrated using the boundary conditions that at $\zeta = 1$, $\partial\Delta_I/\partial\zeta = 0$, $bw^P\Delta^P = 0$, then the result is

$$v^P \equiv \lambda b w^P \Delta^P = \gamma_c(1-\zeta) + \frac{1}{J}\frac{\partial \Delta_I}{\partial \zeta}. \tag{3.5}$$

In terms of the flux variables q^P, v^P defined in (3.3), (3.5) and the momentum flux $m^P \equiv b(w^P)^2$, the governing equations (2.21) to (2.23) and (3.5) become

$$\left.\begin{aligned} &\frac{d}{d\zeta}(q^P)^2 = 2m^P, \quad \frac{d}{d\zeta}(m^P)^2 = 2\frac{v^P q^P}{\lambda},\\ &\frac{1}{J}\frac{d}{d\zeta}v^P = -q^P v^P + \gamma_c q^P(1-\zeta),\\ &\frac{1}{J}\frac{d}{d\zeta}\Delta_I = v^P - \gamma_c(1-\zeta) \end{aligned}\right\} \tag{3.6}$$

with boundary conditions [from (2.24)]

$$q^P = m^P = \Delta_I = 0, \quad v^P = 1 \quad \text{at} \quad \zeta = 0, \tag{3.7}$$

where the reference density for Δ_I has been chosen as that value at $\zeta = 0$. The time dependence of Δ_I is given by (3.2), w_I by (3.3) and u_I by (2.11), (2.12).

A useful alternative set to (3.6) may be obtained by the stretching

$$\eta = J^{\frac{1}{2}}\zeta. \tag{3.8}$$

Then in (3.6) substitute for q^P etc. from (3.9), where

$$\left.\begin{aligned} &q^P = J^{-\frac{1}{2}}q(\eta), \quad m^P = J^{-\frac{1}{2}}m(\eta),\\ &v^P = v(\eta), \quad \Delta_I = J^{\frac{1}{2}}\Delta(\eta) + B(\tau). \end{aligned}\right\} \tag{3.9}$$

Equations (3.6) become

$$\left.\begin{aligned} &\frac{d}{d\eta}q^2 = 2m, \quad \frac{d}{d\eta}m^2 = 2vq/\lambda,\\ &\frac{d}{d\eta}v = -qv + \gamma_c q(1-\eta/J^{\frac{1}{2}}),\\ &\frac{d}{d\eta}\Delta = v - \gamma_c(1-\eta/J^{\frac{1}{2}}). \end{aligned}\right\} \tag{3.10}$$

The equations (3.6) or (3.10) with (3.7) are readily solved for specified γ_c, γ_R and J. Several illustrative examples are given next.

3.1. *A non-diffusive, non-radiating interior*

The solutions obtained by BT can be recovered by taking $J \to \infty$ in (3.6) with $\gamma_c = 1$ and $\gamma_R = 0$. It may be seen that this is a singular limit for the interior buoyancy field (see § 3.3). The solutions to (3.6) in this special case are well represented by the following power series:

$$q^P = \lambda^{-\frac{1}{3}}[\zeta - \tfrac{1}{8}\zeta^2 - \tfrac{3}{160}\zeta^3 - \ldots], \quad m^P = \lambda^{-\frac{2}{3}}[\zeta - \tfrac{3}{8}\zeta^2 - \tfrac{7}{160}\zeta^3 - \ldots]. \tag{3.11}$$

In dimensional variables the plume solutions are

$$\left.\begin{aligned} w^P &= \left(\frac{F_0}{\lambda\alpha}\right)^{\frac{1}{3}}[1 - \tfrac{1}{4}\zeta - \tfrac{9}{160}\zeta^2 - \ldots], \\ b &= \alpha H[\zeta + \tfrac{1}{8}\zeta^2 + \tfrac{1}{20}\zeta^3 + \ldots], \\ \Delta^P &= \left(\frac{F_0}{\lambda\alpha}\right)^{\frac{2}{3}} H^{-1}\left[\frac{1}{\zeta} - \tfrac{7}{8} - \tfrac{2}{320}\zeta - \ldots\right], \end{aligned}\right\} \tag{3.12}$$

and for the interior

$$\left.\begin{aligned} u_I &= -\alpha^{\frac{2}{3}}\lambda^{-\frac{1}{3}}F_0^{\frac{1}{3}}(1-\xi)[1 - \tfrac{1}{4}\zeta - \tfrac{9}{160}\zeta^2 - \ldots], \\ w_I &= -\alpha^{\frac{2}{3}}\lambda^{-\frac{1}{3}}F_0^{\frac{1}{3}}(H/L)[\zeta - \tfrac{1}{8}\zeta^2 - \tfrac{3}{160}\zeta^3 - \ldots] \\ \text{and}\qquad \Delta_I &= \left(\lambda\frac{F_0}{\alpha}\right)^{\frac{2}{3}} H^{-1}\{\tau + \ln\zeta + \tfrac{1}{8}\zeta + \tfrac{11}{640}\zeta^2 + \ldots + l\}. \end{aligned}\right\} \tag{3.13}$$

Here $\zeta = z/H$, $\xi = x/L$ and $\tau = Lt/(\alpha^{\frac{2}{3}}F_0^{\frac{1}{3}})$. The constant of integration l in (3.13*c*) must be chosen by referring Δ_I to a reference density other than at $\zeta = 0$.

These solutions are plotted in figure 8 of BT with $\lambda = 1$. The singular nature of this case with regard to Δ_I and Δ^P is evident. As discussed by BT the action of the turbulent plume in the box gives rise to a stably stratified interior. This is the central feature of the model in general and this case in particular. Several applications of these solutions (actually their three-dimensional equivalents) are given by BT.

3.2. *A diffusive steady state*

The equations describing a diffusive steady state are (3.10) with $\gamma_c = \gamma_R = 0$. The balance of terms in the interior is the classical steady oceanic pycnocline balance between vertical advection of buoyancy and vertical buoyancy diffusion (e.g. Wyrtki 1961). Interpreted in terms of a confined convective system and grossly simplified, the downwelling of cold salty water in the polar regions of the oceans, corresponding to the sinking, negatively buoyant plume of this model when inverted, causes the upward advection of negative buoyancy in the rest of the world's oceans. This in turn is balanced by the downward diffusive erosion of the pycnocline. In this way the mean position of the pycnocline may be held to a fixed depth.

Series solutions for (3.10) in this case are adequate only for small η. The first few terms of the series are

$$q = \lambda^{-\frac{1}{3}}(\eta - \tfrac{1}{30}\eta^3 + \ldots), \quad m = \lambda^{-\frac{2}{3}}(\eta - \tfrac{2}{15}\eta^3 + \ldots),$$
$$v = 1 - \tfrac{1}{2}\eta^2 + \tfrac{2}{15}\eta^4 - \ldots, \quad \Delta = \lambda^{-\frac{2}{3}}(\eta - \tfrac{1}{6}\eta^3 + \tfrac{2}{75}\eta^5 - \ldots).$$

Numerical solutions are displayed in figure 2. The primary difference between these

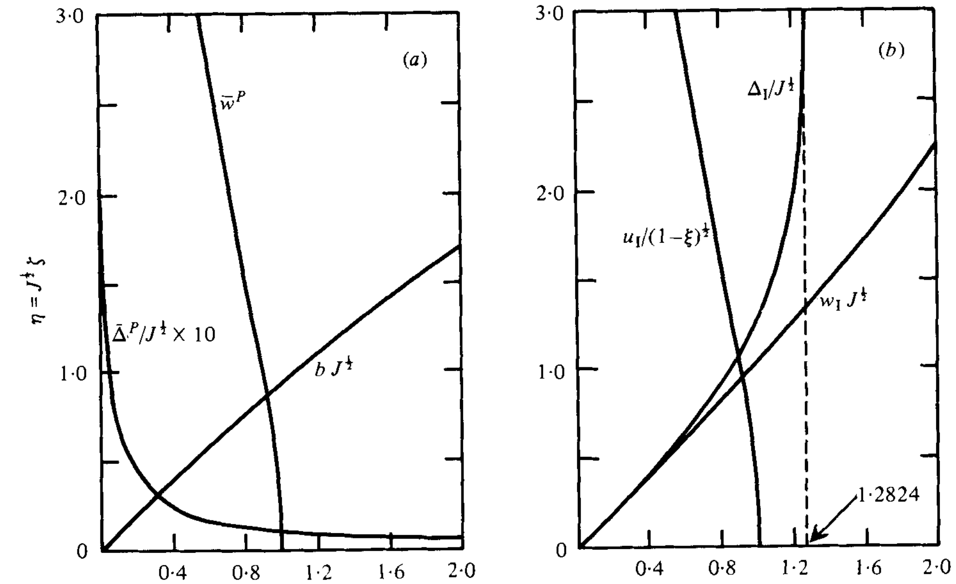

FIGURE. 2 Dimensionless asymptotic solutions: (*a*) for the plume; (*b*) for the diffusive, steady interior. Note that for large η the interior approaches homogeneity with a strong pycnocline below. The maximum η relevant depends on the magnitude of J.

solutions and those of § 3.1 lies in the buoyancy field Δ_I in the interior which now must have a finite gradient at $\eta = 0$ to conduct buoyancy through that surface at a sufficient rate to balance the flux into the plume at 0. Moreover the interior buoyancy field asymptotes to neutral stability with a constant value of $\Delta_I \sim 1{\cdot}2824 J^{\frac{1}{2}}$ as $\eta \to \infty$. Whether or not this asymptote is even approached in a given situation depends on how large J is. (From (3.8) the maximum value of η is $J^{\frac{1}{2}}$.)

The results for this case have been applied and extended to model the deep circulation of the Red Sea (Manins 1973*a*).

3.3. *A weakly diffusive, time dependent interior*

More realistically, the fluid of § 3.1 would be weakly diffusive ($J \gg 1$). Consider the situation where $\eta/J^{\frac{1}{2}} \approx 0$ even for large η. Then equations (3.10) become the boundary layer equations applicable near $\zeta = 0$. Their solutions are, for large η,

$$q \sim (\gamma_c/\lambda)^{\frac{1}{3}} \times \eta, \quad m \sim (\gamma_c/\lambda)^{\frac{2}{3}} \times \eta, \quad v \sim \gamma_c \qquad (3.14a,b,c)$$

and, to within a few per cent,

$$\Delta_I \sim 1{\cdot}28 J^{\frac{1}{2}}(1-\gamma_c) + (\gamma_c + \gamma_R)\tau. \qquad (3.14d)$$

The solutions of § 3.2 and their limit (3.14) are the 'inner' solutions and the non-diffusive solutions of § 3.1 are the 'outer' solutions of a matched asymptotic expansion (Van Dyke 1975) for this problem. To be accurate, the time dependence of Δ_I in (3.14*d*) and (3.13*c*) must be the same. This can be so if for example $\gamma_R = 1-\gamma_c$. A physical realization of this may occur in a model of convection in the lower atmosphere. If the modelled interior is relatively dry except near the lower boundary where there is a strong increase in water vapour concentration then the interior solutions will be unaffected by net radiation absorption. Near the lower 'diffusive' boundary, however, the optical thickness of the air is large and net radiation absorption may be significant there. If in that region $\gamma_R = 1-\gamma_c$ then the 'inner' and 'outer' solutions match in temporal behaviour and the matching rule of matched asymptotic expansions may be applied to the interior buoyancy field to obtain the undetermined constant l in equation (3.13*c*). It is given by

$$\lim_{\eta\to\infty} \Delta_I \text{ [from (3.14}d\text{)]} = \lim_{\zeta\to 0} \Delta_I \text{ [from (3.13}c\text{)] with } J \text{ fixed (large) in both limits.}$$

Then

$$l = 1{\cdot}28(1-\gamma_c)J^{\frac{1}{2}} + \tfrac{1}{2}\ln J. \qquad (3.15)$$

It is now clear that the solutions of BT are the outer solutions of the more general singular perturbation problem specified by (3.6) and (3.7) for large J.

3.4. *A diffusive, radiating, steady interior*

In this steady diffusive system $\gamma_c = -\gamma_R \lesssim 1$ and J is finite. Full solutions, similar to those of § 3.2, are readily obtainable.

Now the mean buoyancy $\bar{\bar{\Delta}}$ at any level is defined as

$$\bar{\bar{\Delta}} = (L\Delta_I + b\Delta^P)/L.$$

$\bar{\bar{\Delta}}$ is the value measured by an averaging instrument passing transversely over a field of line sources. Alternatively it is the value measured by a fixed averaging instrument in a convective field subject to horizontal advection. It can readily be shown that $\partial\bar{\bar{\Delta}}/\partial z$ is always stable and close to $\partial\Delta_I/\partial z$ for the present case and in fact for the general problem formulated in § 2. Thus the action of the unstable plume is to stably stratify the box at any level both in the interior and on average.

4. Concluding remarks

The extended model of turbulent convection from a source in a confined region derived here applies for the following parameter restrictions. (i) The Prandtl number $\sigma \equiv \nu/\kappa \gtrsim 1$ so buoyancy diffusion does not dominate over advection. (ii) The aspect ratio $A \equiv L/H$ must satisfy $A^2 \gg 1$. $A^2 > 1{\cdot}5$ appears to be adequate. Then the interior behaves passively, forced by the boundary layers. (iii) The square of the group $R \equiv \alpha^{\frac{2}{3}}F_0^{\frac{1}{3}}H/\nu$ is the Grashof number characterizing the interior and the turbulent plume above the source of strength F_0, and must satisfy $R \gg 1/\alpha$, $R/A \gtrsim 1$ where $\alpha \ll 1$. These restrictions ensure that the plume is fully turbulent and thin in lateral extent.

The system is steady to $O(\alpha/A)$ with only the buoyancy a linear function of time at every point in the limit of large time. The streamfunction for the interior is then approximately $\psi_I = \text{constant} \times \zeta(1-\xi)$.

Consideration of the outflow region has shown that there the flow is controlled predominantly by inertia–buoyancy dynamics and is similar to the problem of an intrusion into a stratified fluid at high R. Since

$$h/H \approx (\alpha/Fr)^{\frac{1}{2}} \quad \text{and} \quad T_T/T_I \approx (\alpha/Fr)^{\frac{1}{2}},$$

the outflow occupies a small but significant fraction of the box but is able to redistribute plume fluid quickly enough to satisfy the requirements of the lowest-order solutions for the interior.

It may be shown that the corresponding three-dimensional axisymmetric problem must meet the above restrictions [to within a numerical factor of $O(1)$] for validity. The outflow region in that case also has the same scales as found here.

Much of this paper is part of the writer's Ph.D. dissertation (Manins 1973*b*). Sincere thanks are owed to J. Stewart Turner for inspiration and example.

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